BEG THRUST DEDUCTION FRACTION

Plausiblity of the thrust deduction axiom
of the rational theory of ship hull-propeller interaction
based the thrust deduction fraction as function
of loading and displacement influence ratio
according to theory of ideal propulsors in uniform wakes

Start from the thrust deduction fraction as function of loading and velocity ratio

$$1 - \frac{1+\sqrt{1+c}}{\omega + \sqrt{\omega^2 + c}}$$

Simplify denominator

$$1 - \frac{\left(1+\sqrt{1+c}\right)\cdot\left(-\omega+\sqrt{\omega^2+c}\right)}{\left(\omega+\sqrt{\omega^2+c}\right)\cdot\left(-\omega+\sqrt{\omega^2+c}\right)}$$

Expand denominator

$$\left(\omega+\sqrt{\omega^2+c}\right)\cdot\left(-\omega+\sqrt{\omega^2+c}\right) \text{ expand } \rightarrow c$$

Introduce normalised circulation into numerator and simplify

$$\left(1+\sqrt{1+c}\right)\cdot\left(-\omega+\sqrt{\omega^2+\tau^2+2\cdot\tau}\right) \left|\begin{array}{l}\text{substitute}, c = (1+\tau)^2 - 1\\ \text{simplify, assume} = \text{RealRange}(0, 1000)\end{array}\right. \rightarrow -(2+\tau)\cdot\left[\omega - \left(\omega^2+\tau^2+2\cdot\tau\right)^{\left(\frac{1}{2}\right)}\right]$$

Simplify complete expression

$$1 - \frac{-(2+\tau)\cdot\left(\omega-\sqrt{\omega^2+\tau^2+2\cdot\tau}\right)}{\tau^2+2\cdot\tau} \text{ simplify } \rightarrow \frac{-\left[-\tau-\omega+\left(\omega^2+\tau^2+2\cdot\tau\right)^{\left(\frac{1}{2}\right)}\right]}{\tau}$$

Introduce displacement influence ratio

$$\frac{\tau+\chi+1-\sqrt{\chi^2+2\cdot\chi+1+\tau^2+2\cdot\tau}}{\tau} \text{ substitute}, \omega = \chi+1 \rightarrow \frac{\left[\tau+\chi+1-\left(\chi^2+2\cdot\chi+1+\tau^2+2\cdot\tau\right)^{\left(\frac{1}{2}\right)}\right]}{\tau}$$

Arrive at final result

$$\left(\tau^2+2\cdot\tau+1+2\cdot\chi+\chi^2\right) - (\tau+\chi+1)^2 \text{ simplify } \rightarrow -2\cdot\tau\cdot\chi$$

and consequently

$$\frac{\tau+\chi+1-\sqrt{(\tau+\chi+1)^2-2\cdot\tau\cdot\chi}}{\tau}$$

Approximate for smal influence ratio

$$\frac{\tau+\chi+1-(\tau+\chi+1)\cdot\left[1-\frac{\tau\cdot\chi}{(\tau+\chi+1)^2}\right]}{\tau} \text{ simplify } \rightarrow \frac{\chi}{(\tau+\chi+1)}$$

For most purposes this is not precise enough!

Introduce **pressure level** into complete result

$$1-\frac{1+\sqrt{1+c}}{\omega+\sqrt{\omega^2+c}} \text{ substitute}, \omega=\sqrt{1+\sigma} \rightarrow 1-\frac{\left[1+(1+c)^{\left(\frac{1}{2}\right)}\right]}{\left[(1+\sigma)^{\left(\frac{1}{2}\right)}+(1+\sigma+c)^{\left(\frac{1}{2}\right)}\right]}$$

Introduce normalised circulation into numerator and simplify

$$1-\frac{1+\sqrt{1+c}}{\left(\sqrt{1+\sigma}+\sqrt{1+\sigma+c}\right)} \text{ substitute}, c=(1+\tau)^2-1 \rightarrow 1-\frac{\left[1+\left[(1+\tau)^2\right]^{\left(\frac{1}{2}\right)}\right]}{\left[(1+\sigma)^{\left(\frac{1}{2}\right)}+\left[\sigma+(1+\tau)^2\right]^{\left(\frac{1}{2}\right)}\right]}$$

Approximate with respect to normalised Euler number

$$1-\frac{(2+\tau)}{\left[1+\frac{1}{2}\cdot\sigma+(1+\tau)\cdot\left[1+\frac{\sigma}{2\cdot\left(1+2\cdot\tau+\tau^2\right)}\right]\right]} \text{ simplify } \rightarrow \frac{\sigma}{(2\cdot\tau+2+\sigma)}$$

$$\frac{\chi}{\tau+\chi+1} \left| \begin{array}{l} \text{substitute}, \chi=\frac{\sigma}{2} \\ \text{simplify} \end{array} \right. \rightarrow \frac{\sigma}{(2\cdot\tau+2+\sigma)}$$

More important than this approximation are
global approximations of the complete relation.
For this pupose introduce the jet efficiency
instead of normalised loading or circulation

$$\tau\left(\eta_{TJ}\right) := \frac{2}{\eta_{TJ}} - 2$$

$$t\left(\eta_{TJ},\chi\right) := \frac{\tau\left(\eta_{TJ}\right)+\chi+1-\sqrt{\left(\tau\left(\eta_{TJ}\right)+\chi+1\right)^2-2\cdot\tau\left(\eta_{TJ}\right)\cdot\chi}}{\tau\left(\eta_{TJ}\right)}$$

Compute values

$N := 20$

$j := 0 .. N - 2$

$E_{TJ_j} := \frac{j+1}{N}$

$k := 0 .. N - 2$

$X_k := \frac{k+1}{N}$

$T_{j,k} := t\left(E_{TJ_j}, X_k\right)$

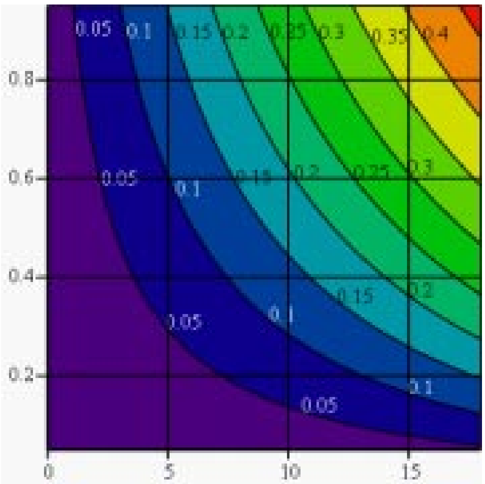
Approximate power law

$A_{j \cdot (N-1)+k,0} := 1$

$A_{j \cdot (N-1)+k,1} := \ln\left(E_{TJ_j}\right)$

$A_{j \cdot (N-1)+k,2} := \ln\left(X_k\right)$

$B_{j \cdot (N-1)+k} := \ln\left(t\left(E_{TJ_j}, X_k\right)\right)$



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Left inverse

$$LI(A) := \left| \begin{array}{l} r \leftarrow rows(A) \\ c \leftarrow cols(A) \\ s \leftarrow svds(A) \\ \text{for } i \in 0 .. c - 1 \\ \quad ISV_{i,i} \leftarrow \left(s_i\right)^{-1} \\ UV \leftarrow svd(A) \\ U \leftarrow submatrix(UV, 0, r - 1, 0, c - 1) \\ V \leftarrow submatrix(UV, r, r + c - 1, 0, c - 1) \\ AIL \leftarrow V \cdot ISV \cdot U^T \\ AIL \end{array} \right.$$

$C := LI(A) \cdot B$

$$C = \begin{bmatrix} -0.505 \\ 1.104 \\ 0.916 \end{bmatrix}$$

$E := A \cdot C - B$

$\text{relative\_error} := \frac{|E|}{|B|}$

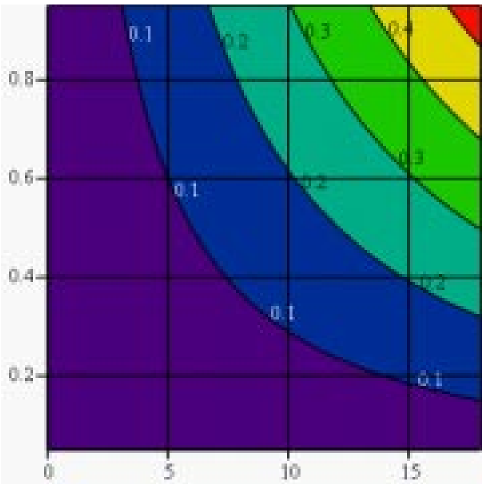
$\text{relative\_error} = 0.024$

$t_{approx}(\eta_{TJ}, \chi) := \exp(C_0) \cdot \eta_{TJ}^{C_1} \cdot \chi^{C_2}$

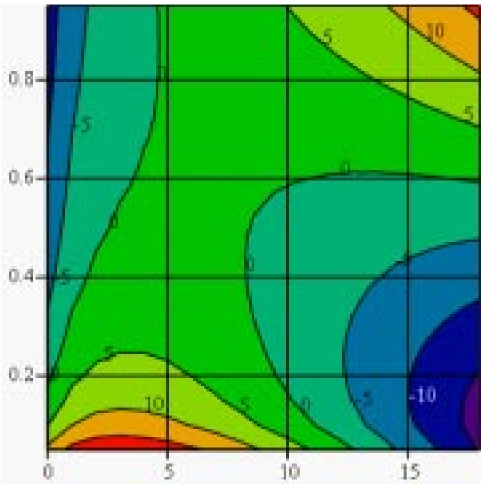
$t_{appr}(\eta_{TJ}, \chi) := 0.603 \cdot \eta_{TJ}^{1.104} \cdot \chi^{0.916}$

$T_{appr_{j,k}} := t_{approx}\left(E_{TJ_j}, X_k\right)$

$\Delta t_{rel_{j,k}} := \frac{T_{appr_{j,k}} - T_{j,k}}{T_{j,k}} \cdot 100$



T_{appr}



Δt_{rel}

Suggested simple approximation

$a_{j \cdot (N-1)+k} := E_{TJ_j} \cdot X_k$

$b_{j \cdot (N-1)+k} := t\left(E_{TJ_j}, X_k\right)$

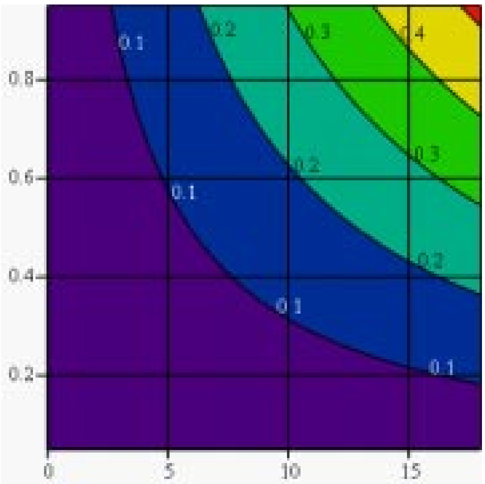
$B := a^T \cdot b$

$A := a^T \cdot a$

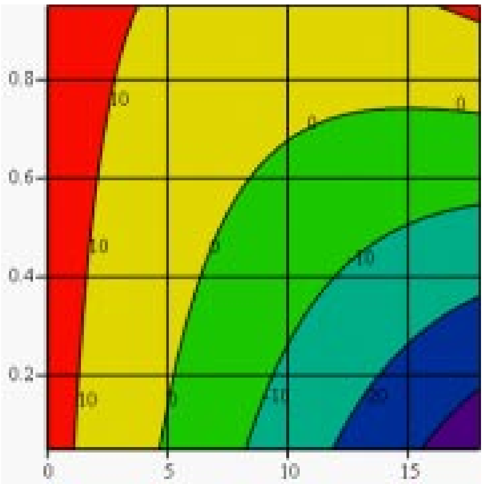
$C := \frac{|B|}{|A|}$ $\qquad C = 0.579748$

$T_{appr_{j,k}} := C \cdot E_{TJ_j} \cdot X_k$ $\qquad$ Every naval architect should know this simple relation!

$$\Delta T\ rel_{j,k} := \frac{T\ appr_{j,k} - T_{j,k}}{T_{j,k}} \cdot 100$$



$T\ appr$



$\Delta T\ rel$

Due to the fact that the jet efficiency
is essentially proportional to the hull advance ratio
these results of the basic theory of ideal propulsors
suggest the thrust deduction axiom
postulating proportionality between
thrust deduction fraction
and hull advance ratio.

In many propulsion experiments with models
the resistance determined via the thrust deduction axiom
has been found to be very nearly the same
as the towing resistance of the bare hull,
provided towing tests were meaningful.

Lecture notes
Hydromechanische Systeme
MS9711271200/9711281800

Translated and upgraded
MS9911161100

END THRUST DEDUCTION FRACTION